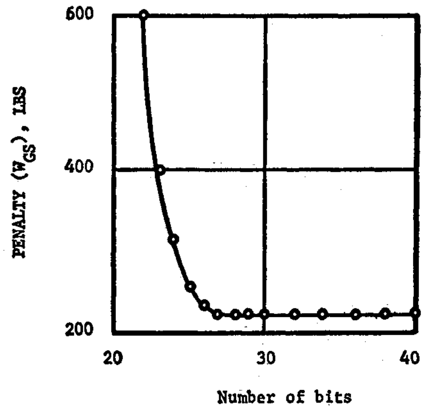

Fig. 4 Plot of penalty, mode 3 (system A) variation with number of computer bits.

a perfect update of velocity, position, and attitude occurs prior to the burn from the parking orbit.

The system parameters used in the evaluation technique can be estimated using techniques that are relatively unsophisticated but are of sufficient accuracy to accomplish the desired result. These techniques were implemented in computer programs now in use at NASA/ERC. The computer programs were exercised on a Jupiter flyby mission. For this mission and the assumptions made, it can be concluded that the accuracy of the strapdown guidance systems evaluated is adequate to accomplish the guidance and navigation of a Jupiter flyby mission.

If the first midcourse correction is made 10 days after launch and the required probability of mission failure attributable to guidance is 0.05, the reliability (mean time to failure) of the pure inertial strapdown guidance systems considered is such that no system examined would succeed if operated continuously from launch to midcourse. If the probability of mission failure attributable to guidance is relaxed to 0.1, the two specified systems still fail if midcourse correction is made 10 days after launch. An optimum system which will succeed is found.

Concentrated attention to reduction of system power requirements would yield a significant reduction in the weight attributable to guidance for this specific mission. This might be achieved by development of a lightweight variable thermal impedance for the ISU.

Reference

[1] Rea, F. G. and Fischer, N. H., "An Improved Method of Estimating Midcourse Fuel Requirements (Approximating the Probability Distribution of the Magnitude of a Vector with Normal, Zero Mean, Components)," paper presented to NASA/ERC Fourth Guidance Theory and Trajectory Analysis Seminar, Cambridge, Mass., May 16–17, 1968.

Far Field Approximation for a Nozzle Exhausting into a Vacuum

John W. Brook*
Grumman Aircraft Engineering Corporation, Bethpage, N. Y.

Introduction

The flowfield generated by a nozzle exhausting into a vacuum is very important to designers of space vehicles and onboard experiments. Many problems arise due to the interaction of the effluent with the vehicle and experiment components. Among these is the generation of cross-coupled torques due to the impingement of reaction control system (RCS) plumes on the exterior of the spacecraft, particularly on solar paddles.

In many cases, the thrust of RCS jets is quite small [e.g., on the NASA-Grumman Orbiting Astronomical Observatory (OAO), thrust ~0.1 lb]; however, the area of the solar paddles is quite large (e.g., 80 ft² on the OAO). The force resulting from plume impingement may act through a large moment arm, possibly causing a control problem, although only a portion of the plume strikes the paddle. The prediction of such forces, in addition to knowledge of the gas-surface interaction properties, requires a reasonably accurate description of density profiles in the far field of the plume (i.e., tens of nozzle-exit radii downstream). This Note describes an analytical method for obtaining these profiles which is more accurate than those presently available.

Discussion and Analysis

The far field of a plume is conventionally described using the continuum method of characteristics. Neglect of kinetic relaxation effects is reasonable, particularly for the density field, if the mass velocity approaches its thermodynamic limit before significant departures from thermodynamic equilibrium occur. Under this assumption, the method may be considered exact. The calculation is time consuming, however, and at large distances from the nozzle, computational difficulties occur. Furthermore, determination of quantities at locations intermediate to the characteristics mesh points requires a multipoint interpolation scheme that adds further computational time.

Several authors[1–3] have attempted to evade the difficulties associated with the method of characteristics by approximate analytical methods directed specifically to the far field. Each of these methods is based on the assumption that the flow at large distances from the exit is radial, i.e., as $r \to \infty$

$$\rho V r^2 = \text{function } (\theta) = F(\theta) = F(0) f(\theta) \qquad (1)$$

where ρ is the density, V the velocity, and r,θ a spherical coordinate system located at the origin (Fig. 1). This assumption has been supported by exact calculations and also follows from the continuity equation. The function $f(\theta)$ is chosen to be a reasonable representation of the expected profiles as well as to be mathematically tractable. Albini[1] expressed $f(\theta)$ in terms of the limiting expansion angle θ_{max} (Fig. 1), based on a correlation of exact calculations, as

$$f(\theta) = \cos^{1/(\gamma-1)}(\pi\theta/2\theta_{max})$$

where λ is the specific heat ratio. However, $F(0)$ was not specified. Roberts[2] and Hill and Draper[3] based their calculations on a one-parameter family of profiles. In Roberts' case,

$$f(\theta) = \cos^k\theta$$

and in Hill and Draper's case,

$$f(\theta) = \exp[-\lambda^2(1-\cos\theta)^2]$$

where k and λ are constants. These constants and the values of $F(0)$ were then obtained by conservation of mass and momentum flux between the nozzle exit and far downstream.

In the latter two cases, however, the values of $F(0)$, which represent the axial density, are low compared with exact calculations. Since mass and momentum flux are conserved, this implies that the density in some region away from the centerline must be higher than the correct value. For applications considered here, where only portions of the

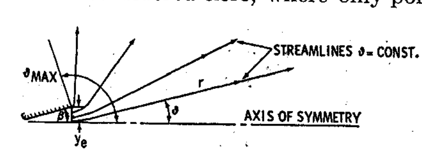

Fig. 1 Schematic of flow from nozzle into a vacuum.

Received February 19, 1969.

* Research Scientist, Research Department. Member AIAA.

plume may be considered, this can result in unacceptable estimates of the impingement forces. A modification of the Hill and Draper profile is presented below which allows matching of the axial density $F(0)$ and conserves mass and momentum flux. However, values of $F(0)$ must be obtained from correlations of previous exact calculations or from a future analysis of the axial density decay rate, so the method is not entirely predictive.

To extend the method further upstream, we rewrite Eq. (1),

$$\rho V r^2 = F(r,0) f(r,\theta) \qquad (1a)$$

where the change of both functions with r is slow. The flow is still radial with origin at the nozzle exit plane. We assume

$$f(r,\theta) = \exp\left[-\lambda^2(r)(1-\cos\theta)^{p(r)}\right] \qquad (2)$$

Using the stagnation density ρ_{stag}, sonic velocity V^*, and exit radius y_e, as density, velocity, and length scales, and assuming radial exit flow and a perfect gas, conservation of mass flux at the exit and at a radius r = const may be written

$$\left[2\left(\frac{2}{\gamma+1}\right)^{-(1/\gamma-1)} A_r F(r,0)\right]^{-1} = I = \int_0^{\theta_{max}} f(r,\theta)\sin\theta d\theta \qquad (3)$$

where $A_r = (y_e/y^*)^2$, the area ratio of the nozzle, and y^* is the sonic radius. Under the same assumptions, and

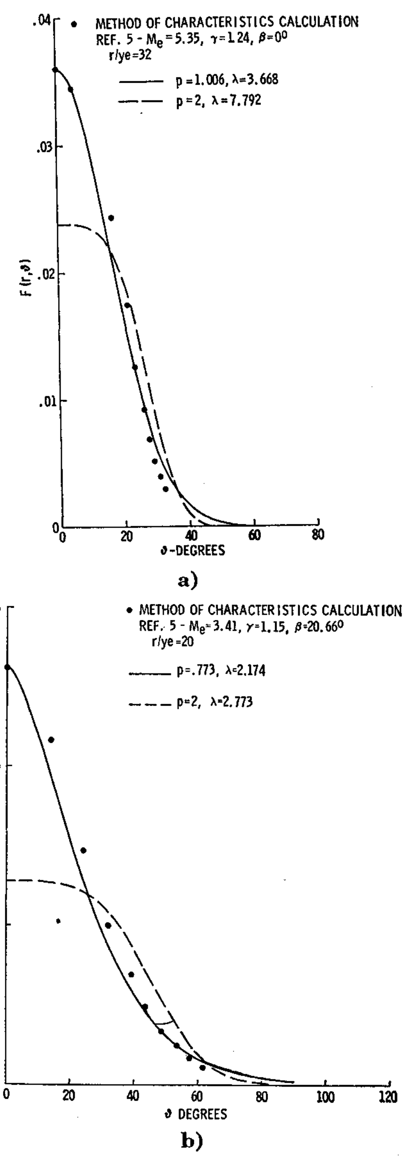

Fig. 2 Comparison of off-axis density profiles.

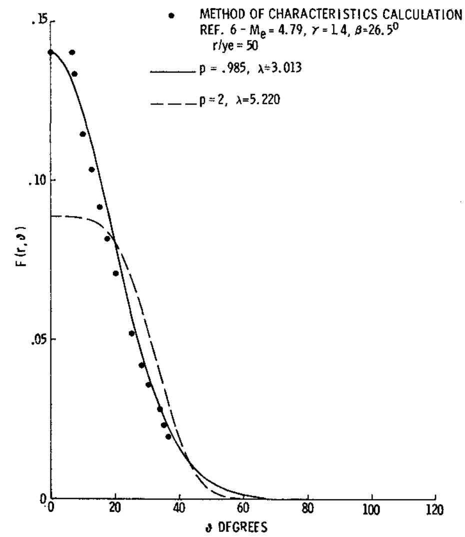

Fig. 3 Comparison of off-axis density profiles—leading characteristic does not reach axis.

expressing the pressure in terms of density, the momentum flux balance may be written

$$C_{FR} V_{max} I F(r,0) = \int_0^{\theta_{max}} \left(\rho V r^2 V + \frac{\gamma+1}{2\gamma}\rho^\gamma r^2\right)\sin\theta\cos\theta d\theta \qquad (4)$$

where

$$C_{FR} = \tfrac{1}{2}(1+\cos\beta)(V_e/V_{max})[1+(\gamma M_e^2)^{-1}] \qquad (5)$$

is the ratio of the thrust coefficient of the nozzle to the maximum thrust coefficient ($= T/T_{max}$); M_e and V_e are the exit Mach number and velocity, respectively, and β is the nozzle lip angle. The effect of nozzle divergence angle, proposed originally by Malina,[4] has been included here. Also, we retain the terms in Eq. (4) which relate to the difference between the velocity on the axis and the maximum velocity, and the local pressure. Previous analyses have ignored these effects. In particular, the nozzle divergence has a substantial effect on the results, whereas the velocity and pressure corrections become important as $\gamma \to 1$ or for moderate values of r.

To calculate the velocity and pressure corrections, we use Eq. (1a) and the isentropic velocity-density relationship. Then if

$$\eta = [(\gamma-1)/(\gamma+1)]^{1/2} F(r,0) f(r,\theta)/r^2 \qquad (6)$$

is assumed small, an expansion of V and ρ in terms of η may be carried out. Substituting the results into Eq. (4), we find

$$\lambda = \left[\frac{\Gamma(2/p)}{\Gamma(1/p)} \frac{1 - h_0\Delta V + h_1(\Delta V)^2}{1 - C_{FR} - e_0\Delta V + e_1(\Delta V)^2}\right]^{p/2} \qquad (7)$$

where Γ is the complete gamma function,

$$h_0 = \gamma^{-(1+2/p)},\ e_0 = \gamma^{-(1+1/p)}$$
$$h_1 = \gamma^{-2/p}A + (2\gamma-1)^{-2/p}(B-A)$$
$$e_1 = \gamma^{-1/p}A + (2\gamma-1)^{-1/p}(B-A)$$
$$A = (\gamma-1)/2 + \tfrac{1}{4},\ B = (\gamma-1)/\gamma$$

and

$$\Delta V = 1 - [(\gamma-1)/(\gamma+1)]^{1/2} V(r,0)$$

Terms $0(\Delta V)^3$ and higher have been neglected in both the numerator and denominator. In Eq. (7) the velocity and pressure corrections are included in ΔV; if ΔV is neglected and p is set equal to 2, the Hill and Draper result (with divergence correction) is obtained. Integration of Eq. (3) leads straightforwardly to

$$F(r,0) = [2/(\gamma+1)] - \frac{1}{\gamma-1} p\lambda^{2/p}/2\Gamma(1/p)A_r \qquad (8)$$

For a given $F(r,0)$, the procedure is to iterate on p, using Eqs. (7) and (8), until the result from Eq. (8) agrees with the given value.

The results of some typical calculations, taken from Refs. 5 and 6, are presented in Figs. 2 and 3 and are compared with exact calculations for various values of γ. Results for $p = 2$, easily obtained during the iteration on p, are included for comparison. The graphs show reasonable agreement with the present model, and indicate that values of p near one are more appropriate than the Hill and Draper case, $p = 2$. Of particular interest is the case presented in Fig. 3. This case is one for which the leading characteristic from the nozzle lip does not reach the axis. For this case the density is known along the entire axis, and the present method is completely self-contained. Despite the fact that the density in the exact case is almost constant near the axis, the present method agrees well with the characteristics calculation.

References

[1] Albini, F., "Approximate Computation of Underexpanded Jet Structure," *AIAA Journal*, Vol. 3, No. 8, Aug. 1965, pp. 1535–1537.

[2] Roberts, L., "The Action of a Hypersonic Jet on a Dust Layer," Paper 63-50, 1963, Institute of Aerospace Sciences.

[3] Hill, J. A. F. and Draper, J. S., "Analytical Approximation for the Flow from a Nozzle into a Vacuum," *Journal of Spacecraft and Rockets*, Vol. 3, No. 10, Oct. 1966, pp. 1552–1554.

[4] Malina, F. J., "Characteristics of the Rocket Motor Unit Based on the Theory of Perfect Gases," *Journal of the Franklin Institute*, Oct. 1940.

[5] Andrews, E. H., Jr., Vick, A. R., and Craidon, C., "Theoretical Boundaries and Internal Characteristics of Exhaust Plumes from Three Different Supersonic Nozzles," TN-D-2650, March 1965, NASA.

[6] Vick, A. R. and Andrews, E. H., Jr., "An Investigation of Highly Underexpanded Exhaust Plumes Impinging Upon a Perpendicular Flat Surface," TN-D-3269, Feb. 1966, NASA.

Operational Flight Program Validation Plan for Missiles

B. E. FERRIER JR.*

Univac, St. Paul, Minn.

MISSILE guidance computer (MGC) systems become increasingly complex as they take over more of the guidance functions. Operational flight programs have been developed to use all of the computational potential a computer has to offer. Support software also has increased in magnitude. As more components are added to a system or program, more tools are required to validate operational success. Knowledge of the mission and system modeling are factors of prime importance in defining and exercising validation techniques. Realism must be built into the flight simulation program to accurately execute all flight functions specified. Operational flight program validation is the verification that the program meets the requirements and the intent of the governing programing specifications. These specifications contain the equations, executive logic, accuracy, and timing requirements to perform all the mission functions required of the MGC programs.

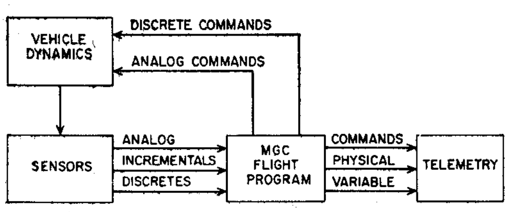

Fig. 1 Computer system interface.

Presented as Paper 68-822 at the AIAA Guidance, Control, and Flight Dynamics Conference, Pasadena, Calif., August 12–14, 1968; submitted August 28, 1968; revision received February 5, 1969.

* Supervising Mathematician, Advanced Systems Department.

Mission Analysis

Significant trajectory events, criteria which initiate or terminate each phase, and special simulation considerations must be identified from mission specifications. The physical process programs (vehicle dynamic modeling) required to simulate vehicle performance must be selected based on a review of the MGC data processing functions on system inputs and the type of MGC outputs and their effects on the vehicle response.

The form of the program to be validated, the acceptance criteria, the number of cases to be simulated, simulation time utilization, and final program quality assurance requirements influence the cost and length of time required to do a job. The acceptance of a program can be influenced by build-up of truncation errors and timing irregularities that are difficult to evaluate during trajectory simulations. Often, data can be analyzed over and over again until a final conclusion is reached that this is the best that can be done with a particular equation because of basic number representation errors and assigned scalings (for fixed-point arithmetic). Careful evaluation at the beginning of the project can minimize this effect by establishing acceptable tolerances. If a program contains logic which cannot be executed by a nominal trajectory, then special or perturbed simulations must be performed.

Computer/System Interface

Interfacing a simulated computer into the total simulation process requires computer input-output (I/O) modeling in detail. Figure 1 shows a typical over-all computer/system interface.

The guidance and control equations use the vehicle dynamic inputs (provided by onboard sensors) and specify the program

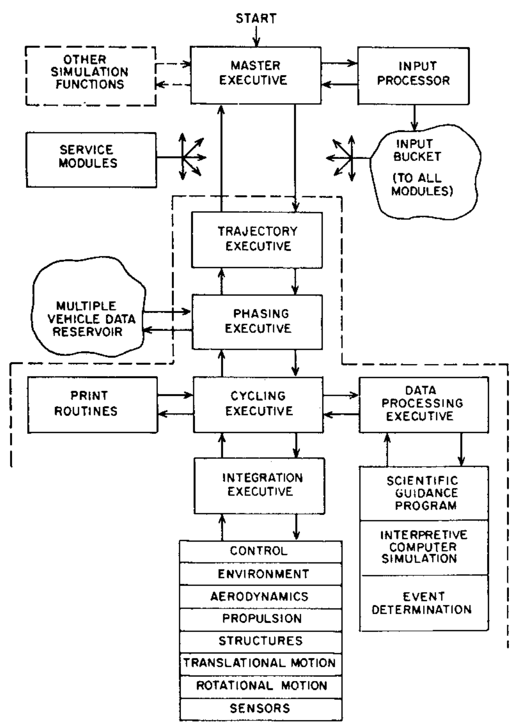

Fig. 2 Module interaction in the flight simulation program.