cutoff, simple models would predict a power low spectrum proportional to $f^{+2.5}$. The shorter baseline experiments at 73.8, 111.5, and 196.5 MHz have made use of the excellent sensitivity and suppression of large angular features to test such models. These data are also still being analyzed but very few, if any, of the sources fit the simple model. One possible interpretation of this discrepancy is that the sources do not consist of a single small diameter component, but rather of a series of more-or-less concentric shell components showing evidence of earlier nuclear activity and subsequent expansion.

V. Conclusion

VLBI techniques find a number of applications at meter wavelengths. Since scattering in the interplanetary medium is known to bias the data, we have planned most of our observations in the antisolar direction. Scattering in the interstellar medium complicates the observation of intrinsic source properties and vice-versa. In the immediate future we will be concerned with analyzing and interpreting the data we have acquired. Once we have a firm understanding of interstellar scattering phenomena and intrinsic source structure in this wavelength region, it will be interesting to turn our attention to interplanetary scattering and to study the phase fluctuations impressed by the interplanetary medium at various baseline lengths and solar elongation angles.

References

[1] C. C. Bare *et al.*, "Interferometer experiment with independent local oscillators," *Science*, vol. 157, pp. 189–191, July 1967.
[2] L. R. Allen *et al.*, "Observations of 384 radio sources at a frequency 158 Mc/s with a long baseline interferometer," *Mon. Notices Roy. Astron. Soc.*, vol. 124, pp. 477–499, 1962.
[3] O. B. Slee and P. K. Wraith, "Discrete radio sources at 38 Mc/s," *Nature*, vol. 214, pp. 971–974, June 1967.
[4] N. W. Broten *et al.*, "Long baseline interferometer observations at 408 and 448 MHz—I. The observations," *Mon. Notices Roy. Astron. Soc.*, vol. 146, pp. 313–327, 1969.
[5] G. W. Brown, T. D. Carr, and W. F. Block, "Long-baseline interferometry of S-bursts from Jupiter," *Astrophys. Lett.*, vol. 1, pp. 89–94, 1968.
[6] G. A. Dulk, "Characteristics of Jupiter's decametric radio source measured with arc-second resolution," *Astrophys. J.*, vol. 159, pp. 671–684, Feb. 1970.
[7] W. C. Erickson *et al.*, "Very long baseline interferometer observations of Taurus A and other sources at 121.6 MHz," *Astrophys. J.*, vol. 177, pp. 110–114, Oct. 1972.
[8] D. E. Harris, G. A. Zeissig, and R. V. Lovelace, "The minimum observable diameter of radio sources," *Astron. Astrophys.*, vol. 8, pp. 98–104, 1970.
[9] N. R. Vandenberg *et al.*, "VLBI observations of the Crab Nebula pulsar," *Astrophys. J. Lett.*, vol. 180, pp. L27–L29, Feb. 1973.

Tracking the Apollo Lunar Rover with Interferometry Techniques

IRVING M. SALZBERG

***Abstract*—Apollo 16 and 17 Lunar Rover position history while transporting astronauts over the lunar surface has been determined to a resolution of better than 1 m and uncertainties of less than 25 m utilizing a specialized very-long baseline interferometry (VLBI) tracking technique. This paper describes the technique, discusses the National Aeronautics and Space Administration worldwide tracking system used to obtain data, discusses the data, and presents results.**

Introduction

Other papers within this special issue treat the historical and scientific aspects of very-long baseline interferometric (VLBI) tracking. This paper will show how these developments have been applied to direct tracking applications.

Traditional metric position determination consists of measuring motion, distance, or angular parameters between a tracking instrument located on the earth's surface and a remote object or target such as an aircraft, a spacecraft, or even a planet. Fundamental types of observations obtained by tracking instruments which will provide these parameters fall into the three basic, but inclusive, categories of mechanical, time, and phase measurements. Electromechanical angular information is obtained from tracking devices by observing the orientation of the movable tracker antenna with respect to a local earth reference while it is viewing the target. Angular measurements of 0.4 mrad are typical with 0.1 mrad, seldom obtainable due to the mechanical nature, size, and radiation pattern of the antenna. Measurement of the time required for an energy pulse, once sent, to be reflected or retransmitted to the tracking instrumentation provides range information assuming knowledge is available concerning propagation time of the transmission media. Pulsed and modulated CW radars operating at C- and S-band frequencies consistently determine slant range within 20 m. Lasers, while they cannot be considered all-weather operational systems, provide range measurements with uncertainties of less than 5 m. Phase measurements obtained from a monochromatic signal source can be measured to within a few hundredths of a cycle of S-band frequencies. In some tracking

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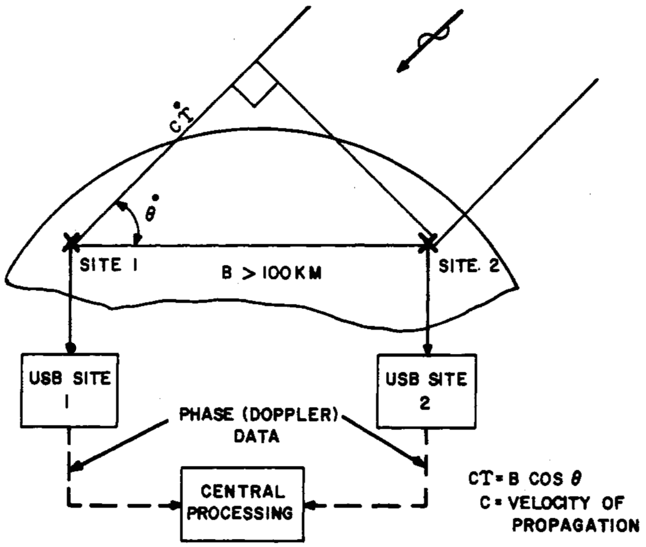

Fig. 1. Monochromatic interferometric tracking.

systems, differential phase measurements of a monochromatic source are made between two receiving antennas separated by several tens of wavelengths providing the basic interferometric angular measurement. The basic accuracy of this system is a function of the frequency used and the distance between antenna receiving elements. Angular accuracies of a few thousandths of an angular degree are readily obtainable with these techniques.

The Apollo 17 manned lunar mission posed a unique tracking problem which could not be solved using any of the conventional techniques described above. Precise information concerning the distance between the moving Lunar Rover Vehicle (LRV) and various scientific packages placed on the lunar surface was required. The LRV navigation system provided information concerning range and bearing to the Lunar Module (LM). However, accuracy of this system was limited due to errors induced by initialization, gyro drift, and quantizing of the readouts. What was desired by the lunar scientists was a complete lunar position history of LRV motion accurate to within 100 m, with individual measurements available at 10- to 30-s time intervals.

Two S-band transmitters functioning at different frequencies were available. The first was a free-running crystal-controlled system feeding an omnidirectional antenna which was located on the LRV and would accompany it on all its lunar-surface travels and transmit directly back to earth a CW signal bearing voice and telemetry modulation. A second transmitter located on the LM received and coherently returned signals originating from the earth during the lunar surface extravehicular activities (EVA). It was the responsibility of the Goddard Space Flight Tracking Network to monitor both of these signals for their telemetry content during the EVA's with one or more of the 14 Unified S-band tracking systems. These earth-based systems are capable of not only stripping voice and telemetry modulation from the S-band carrier but they also provide a count of the difference of S-band cycles received from the number sent to accuracies of 0.05 cycle. Normally this cycle count is converted to Doppler rate and is used for navigating the Apollo vehicle during the cruise to and from the moon. Once the vehicle is safely on the lunar surface these data are not needed for navigation and they are not normally obtained.

To obtain 100-m lunar surface accuracy using conventional techniques implies that the capabilities of earth-bound trackers must exceed 0.25 μrad. However, it was intuitively felt that the Doppler count data could be used in an interferometric sense to obtain the necessary differential separation between the two lunar transmitters. If the locations of the lunar experiment packages could be determined with respect to the LM (and we were assured they could), relative distances between the LRV and LM would meet the Apollo requirements.

A government–industry team consisting of Massachusetts Institute of Technology, Wolf R&D Corp., and Bendix Field Engineering[1] was organized to develop the necessary algorithms and data reduction programs. Techniques, data obtained, and results are discussed in the following paragraphs.

Tracking Technique Development

MIT quickly completed the algorithmic development, the principles of which can readily be understood with pictorial aids. Fig. 1 approximately models the interferometric tracking geometry of this problem. If phase differences between signals received at sites 1 and 2 ($C\tau$) can be measured to an accuracy of 0.07 cycle (or, at S-band frequencies, 10 mm), θ can be determined to 2.5×10^{-9} rad if site separations (B) of 4000 km are available. When computing θ it is necessary to know the value of $C\tau$ *a priori* to less than one S-band wavelength. Without this knowledge the value of $C\tau$ is ambiguous and may be any integral multiple of the signal wavelength plus the additional phase difference determined from the carrier Doppler count. Thus θ cannot practically be determined. However, since the phase measurement made with the Goddard trackers is continuous rate of change, $\dot{\theta}$ can be determined by measuring $C\dot{\tau}$, where $C\dot{\tau}$ is the difference of accumulated phase between site 1 and site 2 over the interval of interest. This interval was 20 s for Apollo 16 and 12 s for Apollo 17. Accuracies of 0.1×10^{-9} rad, or 40 mm/s at lunar distances, should be obtainable with 20-s integration periods.

A minor hardware modification was made to the Goddard trackers to enable them to obtain Doppler data on two separate sources essentially simultaneously which brings us to the second level of our model development depicted in Fig. 2. Determination of $\vec{D}$ in the plane of the paper can be made by subtracting $\dot{\theta}_1$ and $\dot{\theta}_2$ and by having access to appropriate lunar ephemerides assuming that the approximate locations (to within 30 km) of the sources were known. The final step in the model development is to utilize a third tracking site to obtain a two-dimensional vector which, when appropriately transformed, provides two orthogonal selenographic components of $\vec{D}$. Thus it has been demonstrated how differencing Doppler rate data obtained at three independent tracking sites on two transmitters simultaneously provides a measure of the average rate of change of the differential distance between them for a specified interval.

It is known, *a priori*, that the LRV commences its traverse from the LM. Therefore, if it is possible to obtain uninterrupted dual data from three tracking sites it is possible to integrate the rate information and obtain detailed selenographic north and east position information on the LRV rela-

[1] Team representatives included—MIT: Dr. I. Shapiro, Dr. C. Counselman, III, and Dr. H. Hinteregger. Wolf: Dr. C. Martin, J. Vetter, and T. Martin. Bendix: D. Shnidman and E. Shaffer.

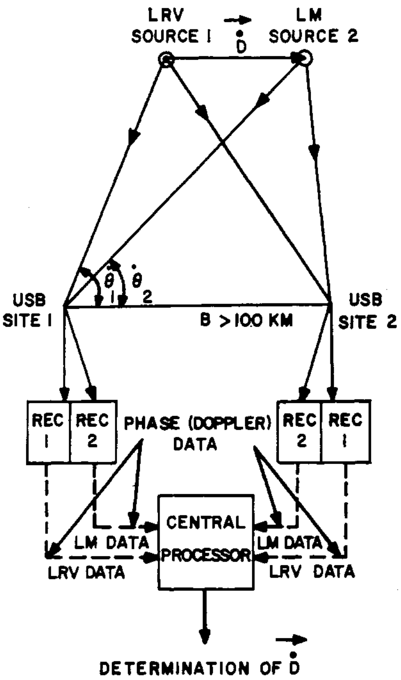

Fig. 2. Double-difference monochromatic VLBI.

tive to the LM. This is exactly what was accomplished for EVA 1 of Apollo 16 and EVA's 1, 2, and 3 of Apollo 17.

It should be noted that the double differencing of Doppler data carries with it several interesting properties. Errors in knowledge of transmitter source location, receiving site locations, earth–moon ephemerides, refraction, and, in general, any systematic errors infiltrating both signals or both sites cancel to the first order, thus providing accuracies even somewhat better than expected. A theoretical discussion of accuracies, however, is complicated by the almost infinite variety of tracking station combinations and their effect on random LRV motion. As a general rule, however, if the tracking triad has good east–west, north–south separation and if the distance between sites exceeds 3000 km, 50-m accuracy can be expected.

Results

Differenced Doppler data were first acquired during Apollo 16 EVA 1 and yielded such excellent results that the technique was used again during Apollo 17 EVA's 1, 2, and 3. To be concise, only the results of Apollo 16 EVA 1, a reasonably typical data set, will be discussed in the following.

Fig. 3 presents, as a function of time, the pertinent events of the LRV traverse for Apollo 16 EVA 1. The traverse began at 20 h, 50 min, 40 s GMT of April 21, 1972 in the Descartes area of the lunar highlands. The initial selenographic location of the LRV was approximately 84 m south and 54 m west of the LM in the area of the Apollo lunar surface experiments package (ALSEP). Madrid, Ascension, and Merritt Island Unified S-band stations were tracking the LRV and LM at the time when the LRV transmitter was switched to the monochromatic transmission mode. Within seconds the LRV departed from the LM heading toward Plum Crater. Arriving at Plum Crater the LRV transmitter was switched from the monochromatic transmission mode and was not reconfigured until seconds prior to the LRV's departure from Plum Crater on a heading towards Spook Crater. After a 25-min stay at Spook Crater the LRV returned to the LM and parked approximately 3 m from it. Redundant Goldstone data were available for the second and third portions of the traverse. The shaded boxes shown in the first and third traverse portions of Fig. 3 are of particular significance. During these tracking periods information obtained from the astronauts indicated that the LRV was stationary. Examination of the integrated individual data points verified that this was the case.

Summarizing the statistics of these two periods in Table I, it is found that noise on the determined north–south, east–west lunar separation distance over a 20-s integration period never exceeded 1 m. These data did indicate, however, that the LRV was moving at about 10 mm/s when in reality it was motionless with respect to the LM. This difficulty is attributed to systematic drifts in the hardware between receivers at one site and was somewhat reduced with hardware modifications for Apollo 17 tracking.

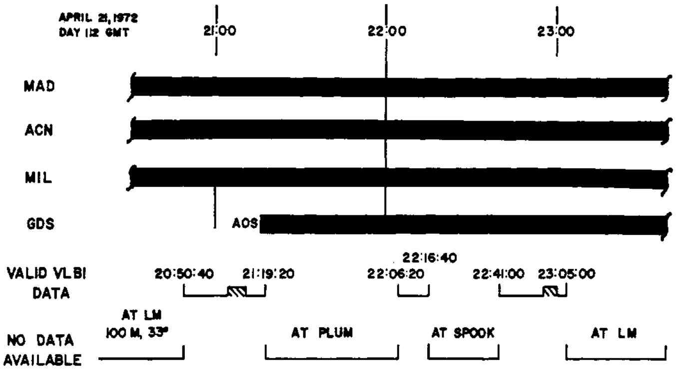

Fig. 3. Events and time line for traverse of EVA 1, Apollo 16.

TABLE I

Noise and Bias of Double-Difference Very-Long Baseline Interferometry (DDVLBI) Data

	Bias (Drift) (mm/s)	Noise (m)
21:06:40 to 21:12:40 (GMT)		
N–S	−8	0.97
E–W	−7	0.40
22:55:00 to 23:01:40 (GMT)		
N–S	−7	0.69
E–W	−4	0.54

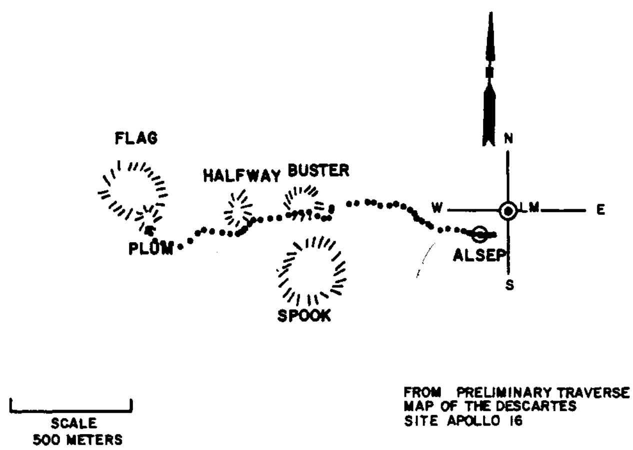

Fig. 4. Lunar Rover traverse on EVA 1 outbound, Apollo 16.

Continuously integrating angular data every 20 s starting from the provided epoch position yields the results shown in Fig. 4 for portion one of the traverse. Similar results are available for the return trip which place the computed parked position of the LRV approximately 10 m from the position indicated by the astronauts.

Based upon the nature and size of the drift rates, noise, and closure figures, a conservative estimate of the LRV position error when using double-difference very-long baseline interferometry (DDVLBI) computation techniques is 25 m over 15-min traverse periods with noise of approximately 1 m.

Conclusions

Differential tracking of two monochromatic sources at lunar distances using differenced Doppler with interferometric data reduction techniques was completely successful and reasonably matched the anticipated performance. Consistent with expectations, Apollo 16 and 17 LRV velocities during lunar traverses have been determined to better than 20 mm/s. These inaccuracies were due to drifting random biases within the tracker instrumentation. The drifting was, in general, random in nature and therefore did not necessarily accumulate range errors when integrating over many minutes of data. It should be mentioned that recent innovations[2] in hardware have clearly shown that even these small systematic biases can be further reduced. Nevertheless, differential range data between the LM and LRV, obtained by integration of rate data, are being provided to lunar scientists with accuracies of 3 to 50 m.

Thus we have now added to our metric tracking techniques a practical new tool which permits the precise measurement of differential motion between two objects located at lunar (and planetary) distances utilizing earth-bound instrumentation and cooperative targets.

[2] C. C. Counselman, III, "Very-long-baseline interferometry techniques applied to problems of geodesy, geophysics, planetary science, astronomy, and general relativity," this issue, pp. 1225–1230.

Spectral-Line Analysis of Very-Long-Baseline Interferometric Data

JAMES M. MORAN

Abstract—Several topics concerning the processing and interpretation of data obtained with very-long-baseline (VLB) interferometers—i.e., tape-recorder interferometers—from spectral-line radio sources are discussed. The instrumental phase shifts are traced through the interferometer. The spectral resolution and normalization of the visiblity spectrum are discussed. Expressions for estimating fringe amplitude, and the associated signal-to-noise ratio, are derived for the case when the integration time exceeds the coherence time of the interferometer. Procedures are presented for measuring the relative spatial distribution of a widely separated collection of point sources utilizing fringe-phase data and rate of change of fringe-phase data.

I. Introduction

A SPECTRAL-LINE interferometer is similar to a conventional interferometer except that the single frequency band is replaced by a set of contiguous frequency bands. The source brightness distribution can thereby be measured as a function of frequency as well as a function of the spatial coordinates and polarization parameters. Very-long-baseline (VLB), or independent tape-recorder, interferometry is additionally complicated by large geometric delays and rates of change of delay, and by the fact that the processed signals are corrupted by phase noise due to limited integration times and to the instabilities of the frequency standards. Also, since it is very difficult to make a single measurement, it is imperative that each measurement contribute as much as possible to the knowledge of the brightness distribution. Four topics related to the processing and interpretation of data from spectral-line radio sources will be discussed: 1) the analysis of frequency-dependent phase shifts in the VLB system, 2) the spectral resolution of spectrometers and the normalization of the fringe-visibility amplitude, 3) the estimation of fringe amplitudes in the presence of noise, and 4) the techniques of mapping a source consisting of a collection of unresolved components at different frequencies.

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II. Phase Shifts in a VLB Interferometer

The essential components of a VLB interferometer are shown schematically in Fig. 1. The signal voltages at frequency $\omega = 2\pi f$ induced in the two receivers by a point source are represented as $A_1(\omega) \exp(j\omega t)$ and $A_2(\omega) \exp[j\omega(t-\tau_g)]$, where $j=\sqrt{-1}$ and τ_g is the excess travel time