interesting to note that the velocity ratio has a definite value in a vacuum when both the pressure ratio and the area ratio are infinitely large. For rockets that operate at high altitudes, not too much additional exhaust velocity can be gained by increasing the exit area ratio above 1000. In addition, the design difficulties and the weight of nozzles with large area ratios make their application impractical.

Example 3–3. Design a nozzle for an ideal rocket that has to operate at a 25 km altitude and give a 5000 N thrust at a chamber pressure of 2.068 MPa (or MN/m^2) and a chamber temperature of 2800 K. Assuming that $k = 1.30$ and $R = 355.4$ J/kg-K, determine the throat area, exit area, throat velocity, and exit temperature.

SOLUTION. At a 25 km altitude, the atmosphere pressure equals 0.002549 MPa using Table 3–2. The pressure ratio is

$$p_2/p_1 = 0.002549/2.068 \equiv 0.001232 = 1/811.3$$

TABLE 3–2. Properties of the Atmosphere

Altitude (m)	Temperature (K)	Pressure Ratio	Density (kg/m^3)
0 (sea level)	288.150	1.0000	1.2250
1,000	281.651	8.8700×10^{-1}	1.1117
3,000	268.650	6.6919×10^{-1}	9.0912×10^{-1}
5,000	255.650	5.3313×10^{-1}	7.6312×10^{-1}
10,000	223.252	2.6151×10^{-1}	4.1351×10^{-1}
25,000	221.552	2.5158×10^{-2}	4.0084×10^{-2}
50,000	270.650	7.8735×10^{-4}	1.0269×10^{-3}
75,000	206.650	2.0408×10^{-5}	3.4861×10^{-5}
100,000	195.08	3.1593×10^{-7}	5.604×10^{-7}
130,000	469.27	1.2341×10^{-8}	8.152×10^{-9}
160,000	696.29	2.9997×10^{-9}	1.233×10^{-9}
200,000	845.56	8.3628×10^{-10}	2.541×10^{-10}
300,000	976.01	8.6557×10^{-11}	1.916×10^{-11}
400,000	995.83	1.4328×10^{-11}	2.803×10^{-12}
600,000	999.85	8.1056×10^{-13}	2.137×10^{-13}
1,000,000	1000.00	7.4155×10^{-14}	3.561×10^{-15}

Source: U.S. Standard Atmosphere, National Oceanic and Atmospheric Administration, National Aeronautics and Space Administration, and U.S. Air Force, Washington, D.C., 1976, NOAA-S/T-1562.

The critical pressure from Equation 3–20 is

$$p_t = 0.546 \times 2.068 = 1.129 \text{ MPa}$$

The throat velocity from Equation 3–23 is

$$v_t = \sqrt{\frac{2k}{k+1} RT_1} = \sqrt{\frac{2 \times 1.30}{1.3+1} 355.4 \times 2800} = 1060 \text{ m/sec}$$

The ideal exit velocity is found from Equation 3–15 and Figure 3–2 using a pressure ratio of 811.3:

$$v_2 = \sqrt{\frac{2k}{k-1} RT_1 \eta} = \sqrt{\frac{2 \times 1.30}{1.30-1} 355.4 \times 2800 \times 0.7869} = 2605 \text{ m/sec}$$

This value can also be obtained from the throat velocity and Figure 3–5. The ideal propellant consumption for optimum expansion conditions is

$$\dot{m} = F/v_2 = 5000/2605 = 1.919 \text{ kg/sec}$$

The specific volume at the entrance to the nozzle equals

$$V_1 = RT_1/p_1 = 355.4 \times 2800/(2.068 \times 10^6) = 0.481 \text{ m}^3/\text{kg}$$

At the throat and exit section the specific volumes are obtained from Equations 3–21 and 3–6.

$$V_t = V_1 \left(\frac{k+1}{2}\right)^{1/(k-1)} = 0.481 \left(\frac{2.3}{2}\right)^{1/0.3} = 0.766 \text{ m}^3/\text{kg}$$

$$V_2 = V_1 \left(\frac{p_2}{p_1}\right)^{1/k} = 0.481(2.068/0.002549)^{0.7692} = 83.15 \text{ m}^3/\text{kg}$$

The areas at the throat and exit sections and the nozzle area ratio A_2/A_t are

$$A_t = \dot{m}V_t/v_t = 1.919 \times 0.766/1060 = 13.87 \text{ cm}^2$$

$$A_2 = \dot{m}V_2/v_2 = 1.919 \times 83.15/2605 = 612.5 \text{ cm}^2$$

$$\epsilon = A_2/A_t = 612.5/13.87 = 44.16$$

This result can also be obtained directly from Figure 3–5 for $k = 1.30$ and $p_1/p_2 = 811.2$. The exit temperature is given by

$$T_2 = T_1(p_2/p_1)^{(k-1)/k} = 2800(0.002549/2.068)^{0.2307} = 597 \text{ K}$$